Probabilistic inference

- Discrete and continuous distributions.

  Let $\mathcal{X}, \mathcal{Y}$ = sets of possible values for respective *random variables* (**rv**'s) x, y.

  Can often assume that x, y are numbers (or vectors).

$$\begin{aligned}
\sum_{x \in \mathcal{X}} P(x) &= 1 \text{ (discrete; } P(x) \text{ is probability distribution)} \\
\int_{x=0}^{\infty} p(x) dx &= 1 \text{ (continuous; } p(x) \text{ is \textit{probability density function} or \textbf{pdf})}
\end{aligned}$$

  NB $P(x), p(x) \geq 0$ but only $P(x) \leq 1$. $p(x)$ can be > 1.

- Conditional and joint probability.

$$\begin{aligned}
\sum_x \sum_y P(x, y) &= 1 \text{ (joint probability distribution for } x \text{ and } y) \\
\sum_y P(x|y) &= 1 \text{ (probability distribution for } x \text{ conditional on } y) \\
P(x, y) &= P(x|y)P(y) \\
&= P(y|x)P(x)
\end{aligned}$$

- Bayes' theorem; prior and posterior distributions; "evidence".

$$P(x|y) = \frac{P(x, y)}{P(y)} = \frac{P(y|x)P(x)}{P(y)}$$

  $P(x)$ is "prior", $P(y)$ is "evidence", $P(x|y)$ is "posterior".

  Dice example: $x \in \{4, 6, 8, 10, 12, 20\}$ is number of faces of die; y is value of a particular roll.

- Some simple graphical models and "truth tables".

Experimental error

- Expectation, variance; moments.

$$\langle x \rangle \equiv E[x] = \sum_{x \in \mathcal{X}} x P(x) \text{ (expectation of } x\text{; two alternative notations)}$$

More generally, can take the expectation of some function $f(x)$

$$\langle f(x) \rangle \equiv E[f(x)] = \sum_{x \in \mathcal{X}} f(x) P(x) \text{ (expectation of } f(x)\text{)}$$

For two alternative probability distributions, $P(x)$ and $Q(x)$, write $\langle x \rangle_P$ and $\langle x \rangle_Q$ to distinguish.

Continuous distributions: substitute $\int p(x) \ldots dx$ for $\sum_{x \in \mathcal{X}}$.

$$\begin{aligned}
\langle x^n \rangle &= \sum_{x \in \mathcal{X}} x^n P(x) \text{ (the ``}n\text{'th moment'' of } x\text{)} \\
V[x] &= \langle x^2 \rangle - (\langle x \rangle)^2 \text{ (variance of } x\text{)} \\
\sigma_x &= \sqrt{V[x]} \text{ (standard deviation of } x\text{)}
\end{aligned}$$

- Rules for $E[x+y]$, $V[x+y]$; moments of $x+y$.

$$\begin{aligned}
E[x+y] &= E[x] + E[y] \\
V[x+y] &= V[x] + V[y] + 2\text{Cov}[x,y] \\
\text{Cov}[x,y] &= \langle xy \rangle - \langle x \rangle \langle y \rangle
\end{aligned}$$

- Approximate error of z when $z = xy$, $z = x/y$, $z = x^y$.